

# HOSSAM GHANEM

## (27) 3.9 Related rates (B)

### Example 6

44 December  
21, 2008

The volume of a cube is increasing at a rate of  $30 \text{ cm}^3/\text{sec}$ . How fast is the total surface area of the cube increasing when the length of each edge of the cube is  $10 \text{ cm}$  long ?

### Solution

$$\frac{dv}{dt} = 30, \quad a = 10$$

$$v = a^3$$

$$\frac{dv}{dt} = 3a^2 \frac{da}{dt}$$

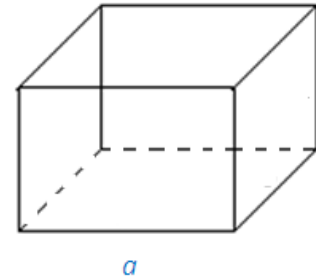
$$30 = 3(10)^2 \frac{da}{dt}$$

$$\frac{da}{dt} = \frac{30}{3(100)} = \frac{1}{10}$$

$$s = 6a^2$$

$$\frac{ds}{dt} = 12a \frac{da}{dt}$$

$$= (12)(10) \cdot \frac{1}{10} = 12 \text{ cm}^2/\text{sec}$$



### Example 7

34 July 22, 2004

A cone of ice cream whose altitude is three times its base radius, is melting, without losing shape, at a rate of  $0.3 \text{ cm}^3/\text{min}$ . Find the rate at which its altitude is changing when its radius is  $2 \text{ cm}$ .

### Solution

$$\frac{dv}{dt} = -0.3, \quad r = 2, \quad h = 3r$$

$$v = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi r^2 (3r)$$

$$v = \pi r^3$$

$$\frac{dv}{dt} = 3\pi r^2 \frac{dr}{dt}$$

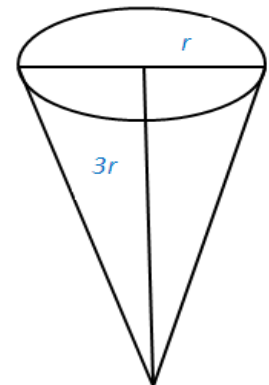
$$-0.3 = 3\pi (4) \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{-0.3}{3\pi (4)} = \frac{-3}{30\pi (4)} = \frac{-1}{40\pi}$$

$$h = 3r$$

$$\frac{dh}{dt} = 3 \frac{dr}{dt}$$

$$\frac{dh}{dt} = 3 \left( \frac{-1}{40\pi} \right) = \frac{-3}{40\pi}$$



**Example 8**47 Dec. ,22  
2009

A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m . If water is being pumped into the tank at rate of  $2 \text{ m}^3/\text{min}$  .Find the rate at which the water level is rising when the water is 3 m deep

**Solution**

$$\frac{x}{2} = \frac{y}{4}$$

$$x = \frac{1}{2} y$$

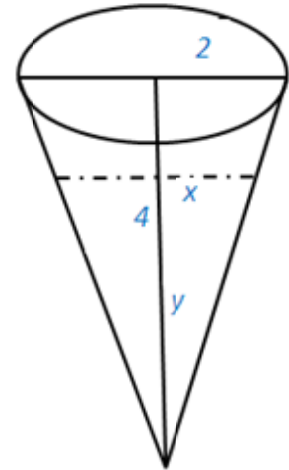
$$v = \frac{1}{3} \pi x^2 y = \frac{1}{3} \pi \cdot \frac{1}{4} y^2 \cdot y$$

$$v = \frac{1}{12} \pi y^3$$

$$\frac{dv}{dt} = \frac{1}{4} \pi y^2 \frac{dy}{dt}$$

$$2 = \frac{1}{4} \pi (3)^2 \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{(4)(2)}{9\pi} = \frac{8}{9\pi}$$

**Example 9**

43 July 19, 2008

A spherical ball made in steel is heated. If the surface area  $A$  of the ball (in  $\text{cm}^2$ ) after Time  $t$  ( in hours ) is given by  $A = \sqrt{t^2 + t - 4}$  . Find the rate at which the radius of ball is changing after four hours.

**Solution**

$$A = \sqrt{t^2 + t - 4}, \quad t = 4$$

$$A = \sqrt{16 + 4 - 4} = \sqrt{16} = 4$$

$$4\pi r^2 = 4$$

$$r^2 = \frac{1}{\pi} \Rightarrow r = \frac{1}{\sqrt{\pi}}$$

$$4\pi r^2 = \sqrt{t^2 + t - 4}$$

$$8\pi r \frac{dr}{dt} = \frac{2t + 1}{2\sqrt{t^2 + t - 4}}$$

$$8\pi \cdot \frac{1}{\sqrt{\pi}} \frac{dr}{dt} = \frac{8+1}{(2)(4)}$$

$$8\sqrt{\pi} \frac{dr}{dt} = \frac{9}{8}$$

$$\frac{dr}{dt} = \frac{9}{64\sqrt{\pi}}$$



**Example 10**

41 July 19, 2007

A point  $P$  moves on the curve  $y = x + 5$  such that  $\frac{dx}{dt} = 3$  units/sec. Find the rate of change of the distance between  $P$  and the point  $Q(2, 0)$  when  $P$  is at  $(-5, 0)$

**Solution**

$$L^2 = (x - 2)^2 + (y - 0)^2$$

$$y = x + 5$$

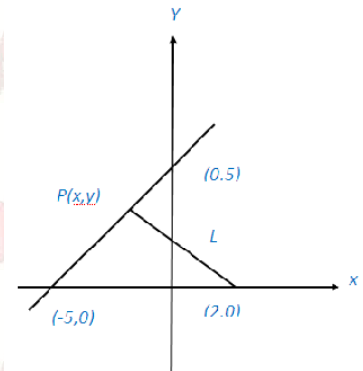
$$L^2 = (x - 2)^2 + (x + 5)^2$$

$$2L \frac{dL}{dt} = 2(x - 2) \frac{dx}{dt} + 2(x + 5) \frac{dx}{dt}$$

$$\text{At the point } (-5, 0) \rightarrow x = -5, \quad y = 0, \quad L = 7$$

$$(2)(7) \frac{dL}{dt} = 2(-7)(3) + 0$$

$$\frac{dL}{dt} = -3$$

**Example 11**52 July 23,  
2011 A

[ 4 Points ] The length  $L$  of a rectangle is decreasing at the rate of 2 cm/sec while the width  $W$  is increasing at the rate of 1 cm/sec. Find the rate of change of the area  $A$  of the rectangle when  $L = 12$  cm and  $W = 5$  cm.

**Solution**

$$\frac{dL}{dt} = -2 \quad \frac{dW}{dt} = 1 \quad L = 12 \quad W = 5$$

$$A = L \cdot W$$

$$\frac{dA}{dt} = L \cdot \frac{dW}{dt} + W \cdot \frac{dL}{dt}$$

$$\frac{dA}{dt} = 12(1) + 5(-2) = 12 - 10 = 2$$



## Homework

9

46 August 1, 2009

A snow ball is melting in such a way that its surface area is decreasing at a rate of  $1 \text{ cm}^2/\text{min}$ . Find the rate of change of its volume when the radius of the ball is  $2 \text{ cm}$

10

27 August 2, 2001

A cone of metal whose altitude is twice its base radius, is heated so that its base radius increases at a rate of  $0.01 \text{ cm}/\text{min}$ . Find the rate at which its volume is changing when the base radius is  $10 \text{ cm}$

11

36 Dec 15, 2005

A plate in a shape of a square is heated. If the area  $A$  of the plate (in  $\text{cm}^2$ ) after time  $t$  (in hours) is given by  $A = \sqrt{1 + t^3}$ . Find the rate at which the sides of the plate is changing after two hours

12

21 May 27, 2001

Sand is falling into a conical pile at a rate of  $2 \text{ m}^3/\text{sec}$ . The height of the cone is always  $\frac{2}{3}$  the radius of its base. Find the rate of change of the radius of the pile when it contains  $48\pi \text{ m}^3$  of sand

13

24 August 3, 2002

The volume  $V$  of a right circular cylinder of radius  $r$  and height  $h$  is changing. If  $r$  is increasing at a rate of  $1 \text{ cm}/\text{sec}$  and  $h$  is decreasing at rate of  $1 \text{ cm}/\text{sec}$ . How fast is  $V$  changing when  $r = 10 \text{ cm}$  and  $h = 6 \text{ cm}$ .

14

26 June 7, 2003

The area of a circular region increases at a rate of  $6 \text{ cm}^2/\text{sec}$ . Find the rate at which its radius changes, when its area is  $9 \text{ cm}^2$ .

15

50 22 December 2010

(4 pts.) A point  $P(x, y)$  moves on the line  $y = 2x - 1$ . So that  $\frac{dx}{dt} = 5 \text{ cm}/\text{sec}$ . Find the rate at which the distance between  $P$  and  $Q(-1, 1)$  is changing when  $P$  is at  $(0, -1)$

16

51 8 May 2011

[3 pts.] A point  $P(x, y)$  is moving on the curve  $y = x^2 - 3$ , such that the  $x$ -coordinate is increasing at rate of  $7 \text{ cm}/\text{sec}$ . what is the rate of change of the  $y$ -coordinate when the point  $P$  reaches  $(3, 6)$ ?